

Hello AP Calculus Students!

I am so excited to have the opportunity to get to know all of you and spend the year in the amazing world of Calculus.

This summer I have one thing that I want both AB and BC to complete, and that is the pdf document that I have attached here. This document is a review of much needed information from PreCalculus and Algebra 2. Now, I am very aware of how easy most of these would be to just scan with google lens. However, I am hoping you choose the high road and actually work through the problems. We will have a quiz over the problems the first couple of days of school.

Second - if you are in BC I would like you to join the Khan Academy class and go through the Limits unit. This unit is one that we will kick off with but will move very fast through and having a strong understanding before the year starts will be helpful. AB students are welcome to look through it as well, but not required.

Khan Academy - <https://www.khanacademy.org/join/F3RTN7JB>

If anyone would be interested in some in person review sessions before school kicks back I will give more information for those in the middle of July. (everything will depend on the floor construction at school).

Have a great summer and see you in August!

Mrs. Mahoney

AP Calculus Summer Assignment

1. Simplify: a) $\frac{x^3-9x}{x^2-7x+12}$ b) $\frac{x^2-2x-8}{x^3+x^2-2x}$ c) $\frac{x^2-9}{x^2+4x-3}$
2. Rationalize the denominator: a) $\frac{2}{\sqrt{3}+\sqrt{2}}$ b) $\frac{4}{1-\sqrt{5}}$
3. Solve for n: a) $4 + 3(n-1) = 511$ b) $5 \times 2^{n-1} = 20480$ c) $\frac{n(5+3n)}{2} = 175$ d) $\frac{1}{1-4n} = 2$
4. Write the expression as a logarithm of a single quantity: a) $\ln 5 + \ln(x^2-1) - \ln(x-1)$
b) $2 \log_{10} \sqrt{x} + 3 \log_{10} x^{1/3}$
5. Solve for x (without a calculator): a) $2^x = 64$ b) $10^x = 1000$ c) $\left(\frac{x}{2}\right)^3 = 125$
d) $x^5 = 0.00001$ e) $\frac{1}{3} = 3^{2x+2}$ f) $5^{(x+1)} = 25$ g) $\ln e^3 = x$
h) $\ln e^x = 4$ i) $\ln x + \ln x = 0$ j) $e^{\ln 5} = x$ k) $\ln 1 - \ln e = x$
l) $\ln 6 + \ln x - \ln 2 = 3$ m) $\ln(x+5) = \ln(x-1) - \ln(x+1)$
6. Express in terms of x (solve for y): a) $\log y = x + 2$ b) $\ln y = 2 \ln x$
7. Solve for x: a) $(x-2)(x+2)(x-1)^2 = 0$ b) $\sqrt{x-5} + 1 = 3$
8. Factor completely: a) $x^6 - 16x^4$ b) $4x^3 - 8x^2 - 25x + 50$ c) $8x^3 + 27$
d) $x^4 - 1$ e) $x^3 + 4x^2 - 9x - 36$
9. Find all solutions: a) $x^6 - 16x^4 = 0$ b) $4x^3 - 8x^2 - 25x + 50 = 0$ c) $x^2 = 23 + 2x$
d) $2x^2 - 3x + 3 = 0$
10. Solve the equations: a) $4x^2 + 12x + 3 = 0$ b) $2x + 1 = \frac{5}{x+2}$ c) $\frac{x+1}{x} - \frac{x}{x+1} = 0$
11. Solve for x: a) $|-x+4| \leq 1$ b) $|5x-2| = 8$ c) $|2x+1| = x+3$
12. Given $f(x) = x-3$ and $g(x) = \sqrt{x}$, find the following: a) $f(g(x))$ b) $g(f(x))$ c) $f(f(x))$
13. Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x+3}$. Find a rule for $f(g(x))$ and give the domain of the composite function.
14. Use long and synthetic division to find the remainders of: a) $x^3 - 3x^2 + 5$ by $x-3$
b) $x^5 - 4x^4 + x^3 - 7x + 1$ by $x+2$
15. Is $x = -1$ a zero of the function $P(x) = x^3 - 3x^2 - x + 3$? Why or why not?
16. Find the inverses of the functions: a) $f(x) = 2x + 3$ b) $g(x) = \frac{1}{2x-5}$
17. a) Given $f(x) = |x-3| - 5$, find $f(1) - f(5)$. b) Given $f(x) = x^2 - 3x + 4$, find $f(x+2) - f(2)$.

18. If $f(x) = 3x^2 - x - 1$, find $f(0)$, $f(-2)$ and $f(i)$.

19. Determine the equations of the following lines: a) the line through $(-1,3)$ and $(2,-4)$
 b) the line through $(-1,2)$ and perpendicular to the line $2x-3y+5=0$
 c) the line through $(2,3)$ and the midpoint of the line segment from $(-1,4)$ to $(3,2)$
 d) the line through $(2,4)$ and parallel to the line $2x+3y-8=0$
 e) the line with slope of 5 that passes through the point $(-1,3)$ and intersects the x-axis at a point. What are the coordinates of this point?

20. Find the vertex and axis of symmetry for the parabola $y = 2x^2 + 8x + 5$.

21. Test the equation $x^2 + 4y^2 = 1$ for symmetry about the x-axis, the y-axis, the line $y=x$, and the origin.

22. Show work to determine if the following relations are even, odd, or neither: a) $f(x) = 2x^2 - 7$
 b) $f(x) = -4x^3 - 2x$ c) $f(x) = 4x^2 - 4x + 4$

23. a) Find the domain of the function $f(x) = \frac{3x+1}{\sqrt{x^2+x-2}}$. b) Find the domain and range of the functions:
 $f(x) = 7$ and $g(x) = \frac{5x-3}{2x+1}$.

24. a) The graph of a quadratic function (a parabola) has x-intercepts of -1 and 3 and a range consisting of all numbers less than or equal to 4 . Determine an expression for the function.

b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$.

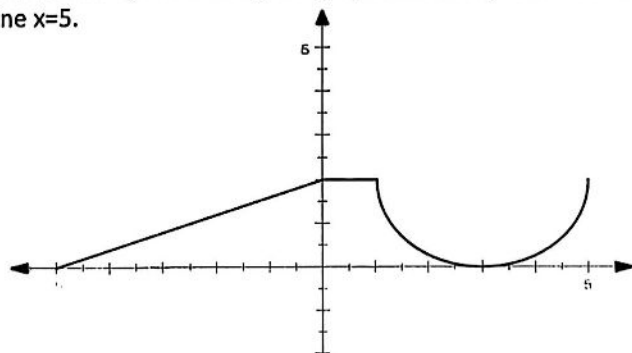
25. Sketch the graph of the functions: a) $f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$

b) $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x+3 & [2, \infty) \end{cases}$

c) $g(x) = |3x + 2|$

d) $h(x) = |x(x-1)|$

26. a) Write a piecewise function that is represented by the graph below. b) Find the area bounded by the graph, the x-axis, and the line $x=5$.



27. Use x-intercept, y-intercept, vertical asymptote, horizontal asymptote, holes, and end behavior to sketch the graph of the function $f(x) = \frac{x^2+6x+5}{x^2-x-2}$.

28. Sketch the graph of the circle $x^2 + (y - 2)^2 = 25$. Find the circumference and area of the circle.

29. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$.

a) $f(x) - 4$ b) $f(x - 4)$ c) $-f(x + 2)$ d) $5f(x) + 3$ e) $f(2x)$ f) $|f(x)|$

30. Without using a calculator, evaluate the following: a) $\sin \frac{5\pi}{4}$ b) $\cos 210^\circ$ c) $\tan^{-1}(-1)$

d) $\sin^{-1}(-1)$ e) $\cos^{-1}(-1)$ f) $\cos \frac{9\pi}{4}$ g) $\sin^{-1} \frac{\sqrt{3}}{2}$ h) $\tan \frac{7\pi}{6}$

31. Find the solutions of the equations for $0 \leq x \leq 2\pi$ (unless otherwise specified):

a) $2\sin^2 \theta = 1 - \sin \theta$ b) $2 \tan \theta - \sec^2 \theta = 0$ c) $\sin 2\theta + \sin \theta = 0$
d) $\sin x^2 - \sin x = 0$ e) $3\sin^2 x = \cos^2 x$ f) $\cos^2 x - \sin^2 x = \sin x, (-\pi, \pi]$

32. If $\tan \theta = \frac{5}{2}$ and $\pi < \theta \leq \frac{3\pi}{2}$, find: a) $\csc \theta$ b) $\sec \theta$ c) $\cot \theta$

33. Find the exact value of $\sin \left(\cos^{-1} \left(-\frac{1}{3} \right) \right)$.

34. Prove: a) $\tan^2 \theta - \tan^2 \theta \sin^2 \theta = \sin^2 \theta$ b) $\sin \theta + \cot \theta \cos \theta = \csc \theta$
c) $\tan x + \cot x = \sec x \csc x$ d) $\frac{1}{\tan^2 x + 1} = \cos^2 x$

35. Find $\frac{f(x+\Delta x) - f(x)}{\Delta x}$ for each function: a) $f(x) = 8x^2 + 1$ b) $f(x) = x^2 - 2x - 3$

36. Find $\frac{f(x+h) - f(x)}{h}$, given the functions: a) $f(x) = \frac{1}{x+1}$ b) $f(x) = 2x + 3$ c) $f(x) = x^2$

37. Simplify the expression: $f(x) = \frac{2(x+h)^2 + 1 - (2x^2 + 1)}{h}$. Then find $\lim_{h \rightarrow 0} f(x)$

38. Find the value of $\sum_{k=1}^4 \left(\frac{1}{3^k} \right)$.

39. a) Let $x_1 = 6$, $x_2 = 8$, $x_3 = 9$ and $x_4 = 13$. Evaluate $m = \frac{\sum_{i=1}^4 (x_i)}{4}$.

b) Then use the values above and m to evaluate $\frac{\sum_{i=1}^4 (x_i - m)^2}{4}$.

40. A water tank has the shape of a cone. The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep, what is the surface area of the top of the water?

41. Two cars start moving from the same point. One travels south at 100 km/hr, the other west at 50 km/hr. How far apart are the two cars after 2 hours?

42. A kite is 100 m above the ground. If there is 200m of string let out, what is the angle between the string and the horizontal? (Assume the string is perfectly straight).



43. Find the surface area of a box of height h whose base dimensions are p by q , and that satisfies the each condition: a) the box is closed. b) the box has an open top. c) the box has an open top and square base with side length p .

44. A seven foot tall ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression (in terms of x) for the distance from the foot of the ladder to the base of the wall.

45. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent in the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x .

46. An aluminum cube measures 12 cm on each edge. This cube is melted down and reshaped to form a right circular cone with a diameter of 12cm. Find the height of the cone.

47. Suppose you have 100 yards of fencing with which to enclose an area. Think of 3 different shapes you could form using all the fencing. Find the area of each of the three shapes, and show work, including diagrams. Determine which shape maximizes the area.

48. Consider a right circular cone with a diameter of 24cm for the base and an altitude of 32 cm. If the cone is held with its vertex down and water is placed into it until it is half full, what is the depth of the water?



49. Mr. Mealor lives 22 miles from ACS. He drives at a rate of 60mph to school, but on the way home the weather is bad and he drives at a rate of 30mph. What is Mr. Mealor's average speed for the whole trip? (Hint the answer is not 45mph).

50. Enjoy your summer!

